Tree Decomposition

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RESEARCH - UFZ

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Parallelization

Distribution of calculations onto multiple computational units, so parts of the calculations can be done simultaneously.

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	1	2	3	5	8	13
+	13	8	5	3	2	1

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Distribution of calculations onto multiple computational units, so parts of the calculations can be done simultaneously.

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Parallelization

Distribution of calculations onto multiple computational units, so parts of the calculations can be done simultaneously.

	1	2	3	5	8	13
+	13	8	5	3	2	1
	14		8		10	

Profit: in the optimal case the new calculation time is the serial time by the number of computational units.

Parallelization

A cheese cake needs about 18 hours for baking. How much time is needed to bake the cheese cake with the same result if there where 6 ovens available?

Parallelization

1 2 3 5 8 13

Parallelization

Fibonacci sequene (shifted by 1)

1 2 3 5 8 13

Parallelization

Fibonacci sequene (shifted by 1)

1 2 3 5 8 13 21

Parallelization

Fibonacci sequene (shifted by 1)

1 2 3 5 8 13 21 34

Parallelization

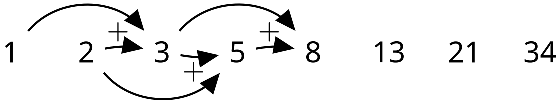
Fibonacci sequene (shifted by 1)

Parallelization

Fibonacci sequene (shifted by 1)

Parallelization

Fibonacci sequene (shifted by 1)



Parallelization

Pascal triangle

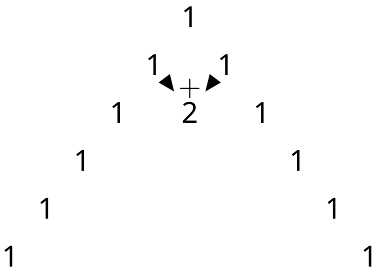
Parallelization

Pascal triangle

					1					
				1		1				
			1				1			
		1						1		
	1								1	
1										1

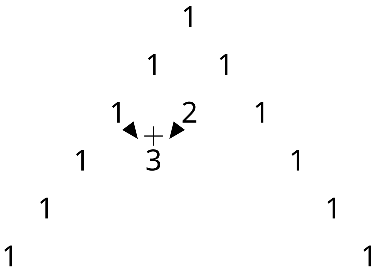
Parallelization

Pascal triangle



Parallelization

Pascal triangle



Parallelization

Pascal triangle

1

1 1

1 2 1

\+

1 3 3 1

1 1

1 1

Parallelization

Pascal triangle

1

1 1

1 2 1

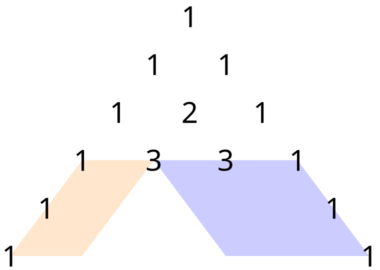
1 3 3 1

1 1

1 1

Parallelization

Pascal triangle



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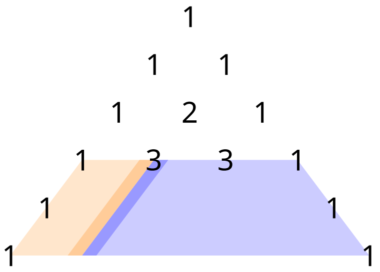

Parallelization

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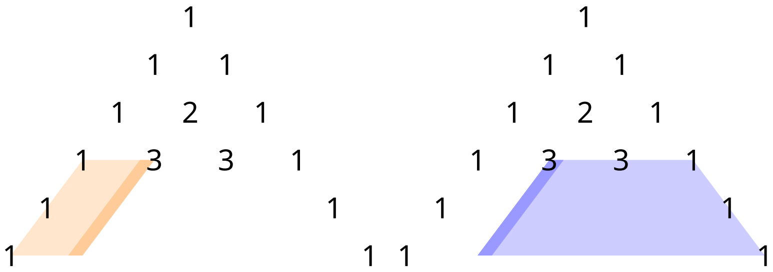
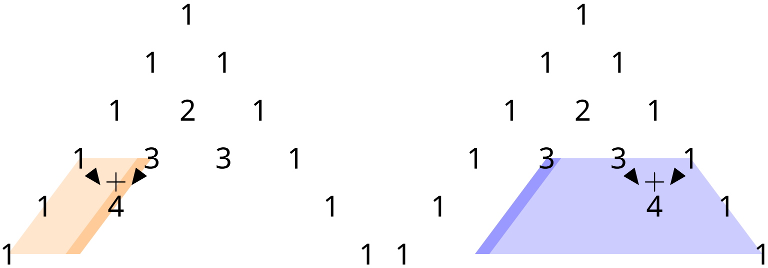


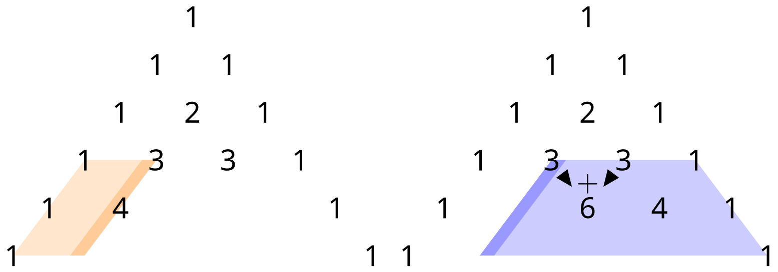
Parallelization

Pascal triangle



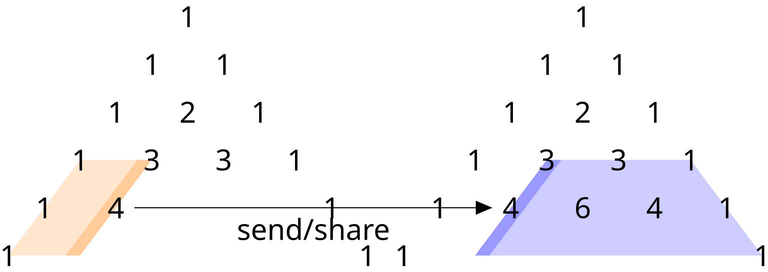
Parallelization

Pascal triangle



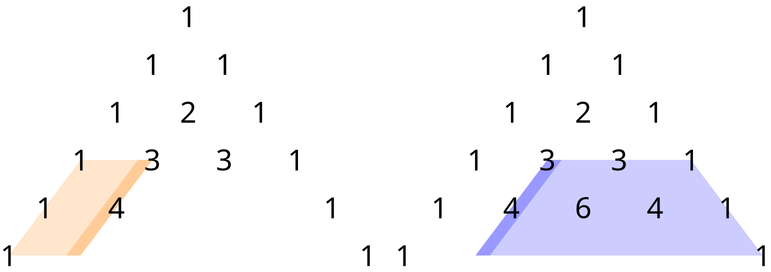
Parallelization

Pascal triangle



Parallelization

Pascal triangle

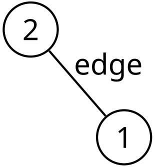


Graphs

vertex

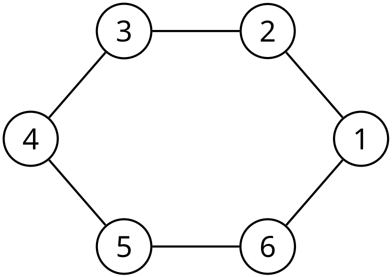
graph

Graphs



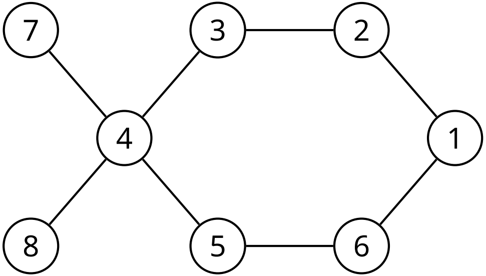
Graph

Graphs



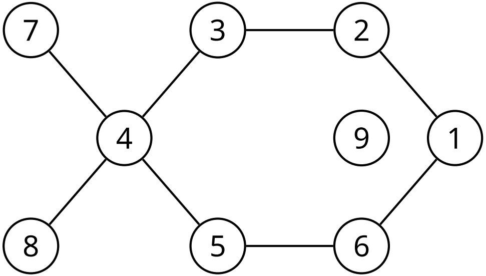
Cycle (nevertheless a graph)

Graphs



no cycle, but a connected graph

Graphs



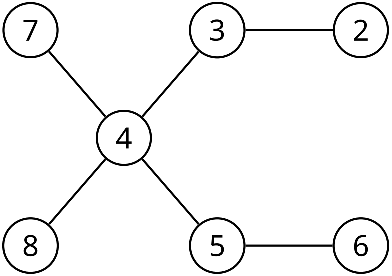
an unconnected graph

Trees

A tree is acyclic connected graph

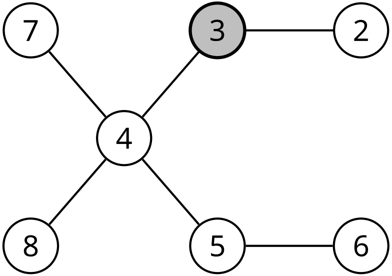
Trees

A tree is acyclic connected graph



Trees

A tree is acyclic connected graph



Trees

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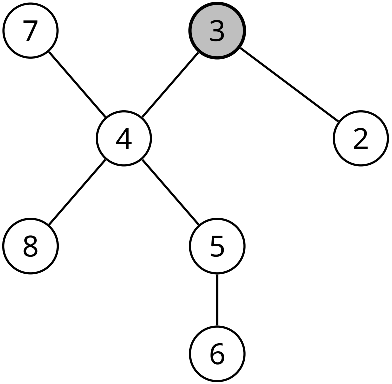

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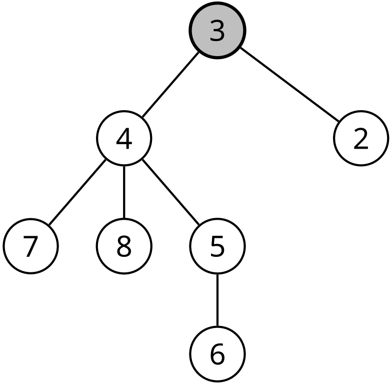

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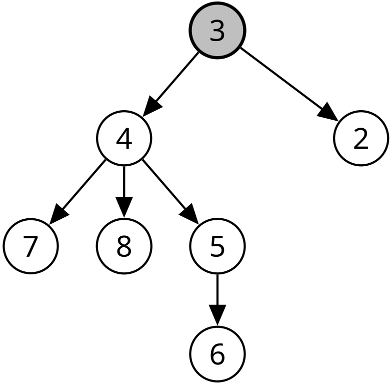
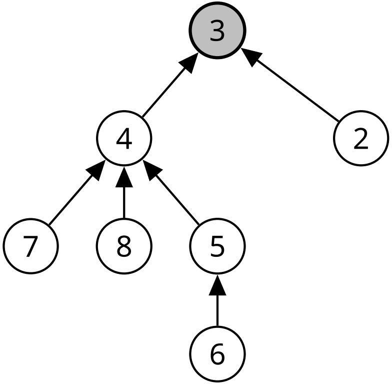


gewurzelter Baum, Out-tree

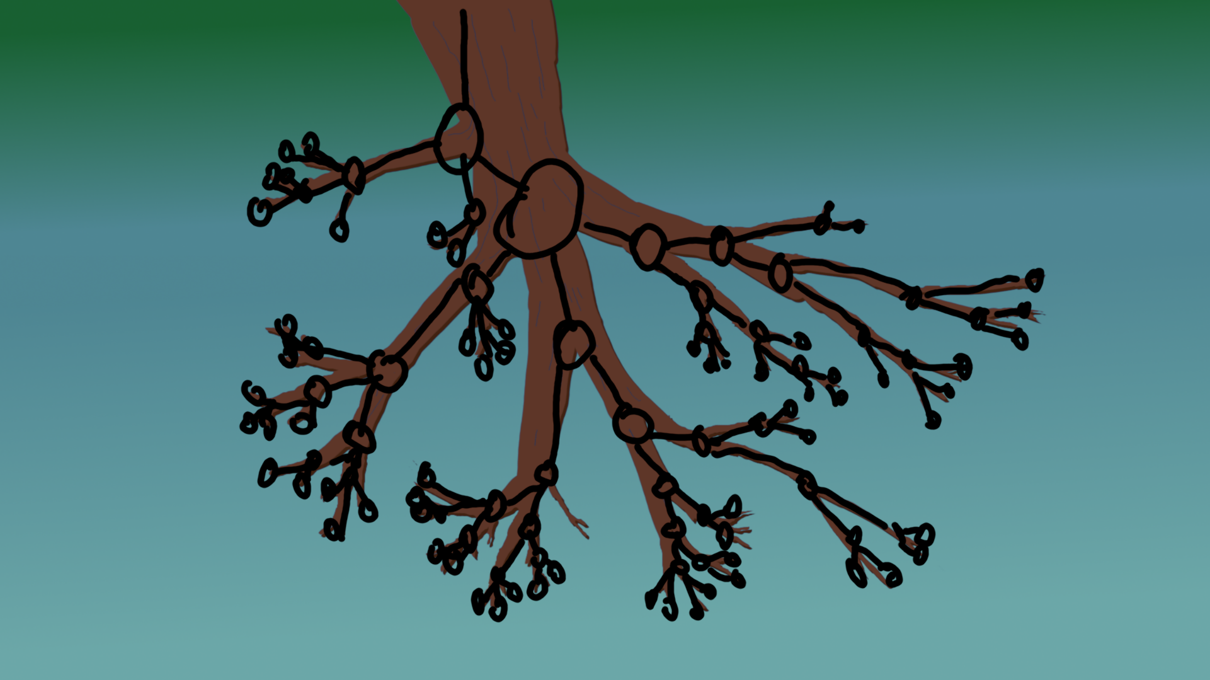
Trees

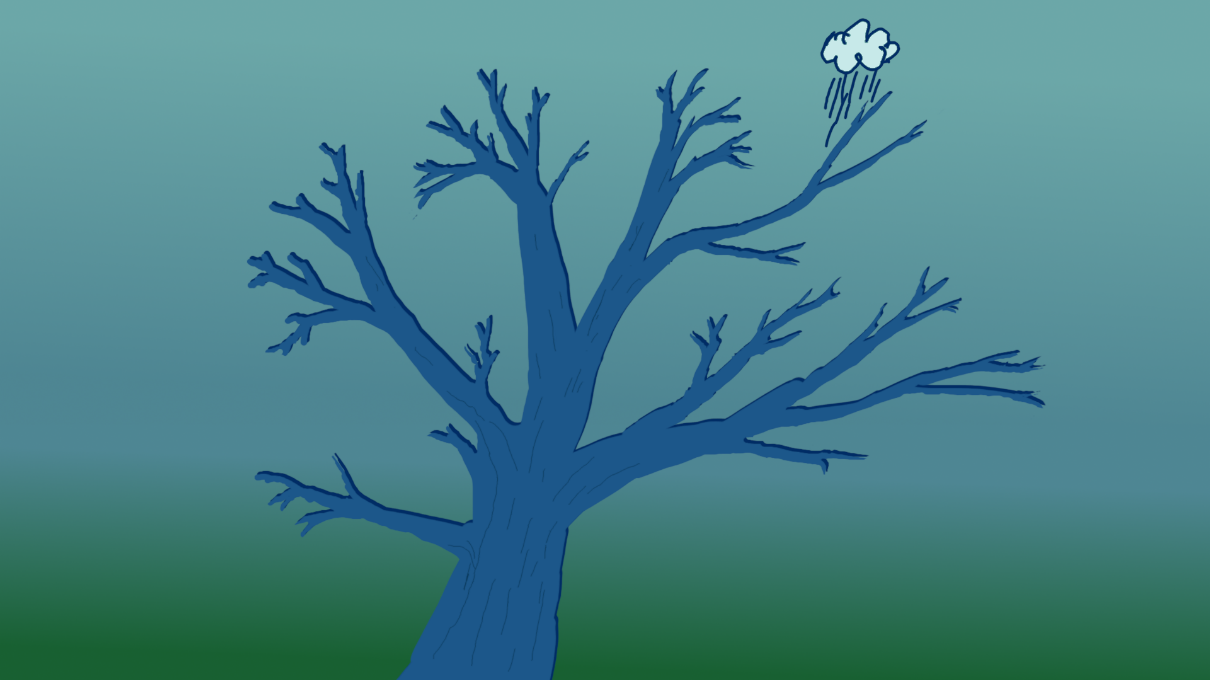
A tree is acyclic connected graph

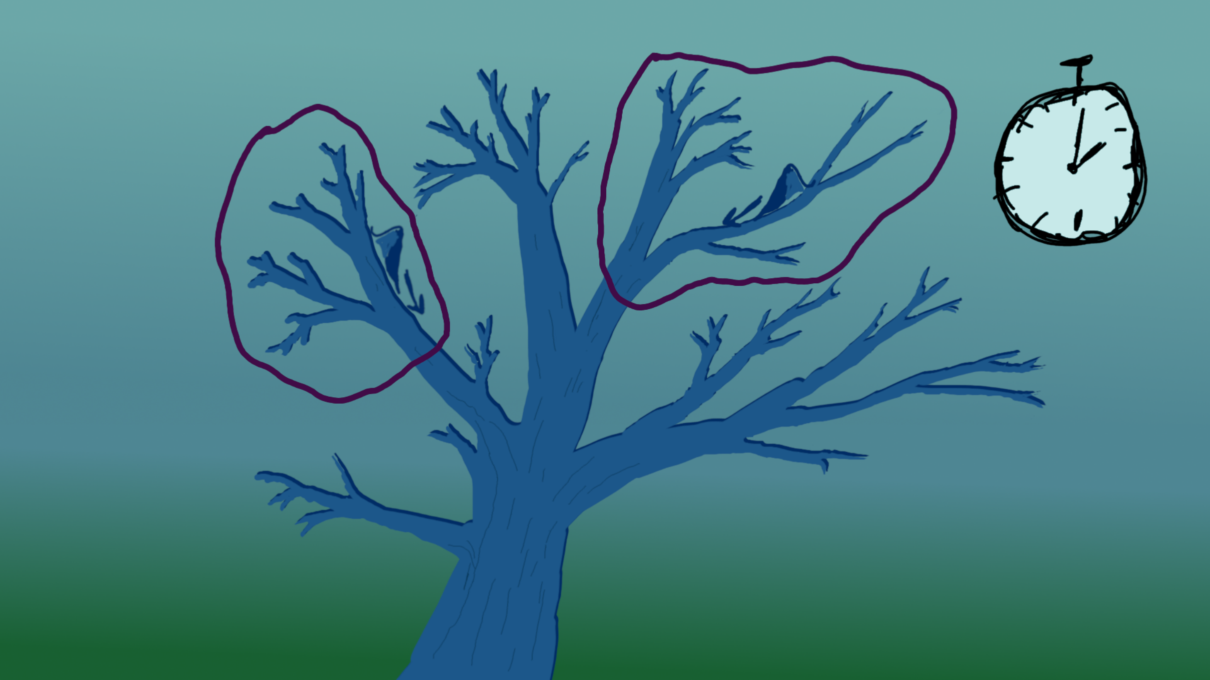


gewurzelter Baum, In-tree

rivernetwork (Hydrology)







Tree Decomposition

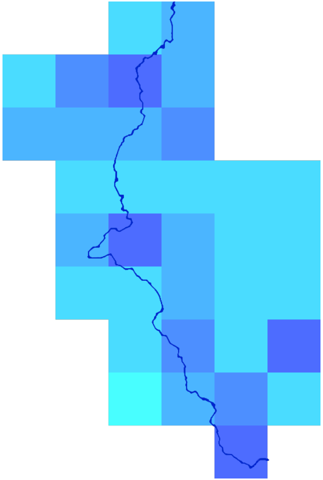
Tree Decomposition

The idea is to decompose the tree into subtrees and distribute these onto computing nodes.

- the case of dynamic distribution of subtrees onto the nodes has been studied [Li et al., 2011]
- we discuss a static distribution

Tree Decomposition

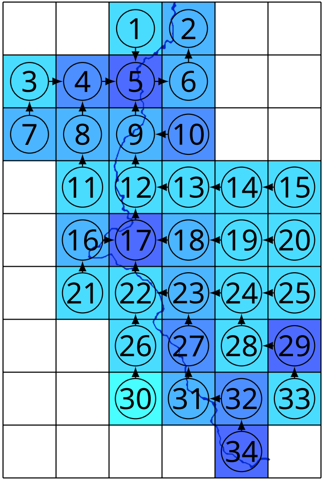
Example: Testbasin



As test basin we use the Moselle with 34 cells.

Tree Decomposition

Example: Testbasin



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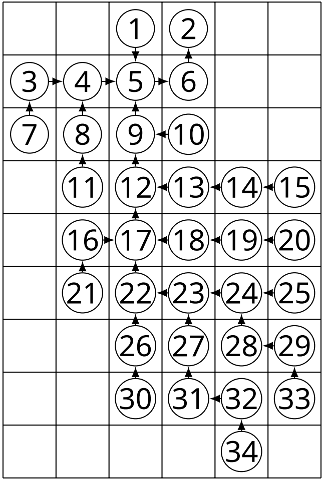
Tree Decomposition

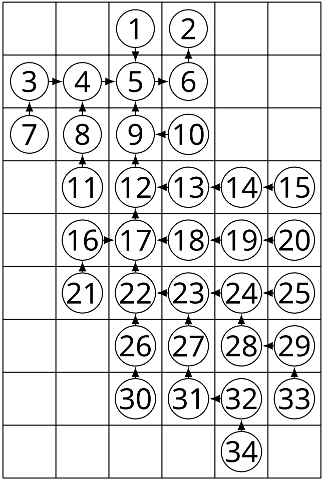
Example: Testbasin



As test basin we use the Moselle with 34 cells.

Tree Decomposition

Goal



Cut of subtrees with nice sizes recursively and distribute them onto the computing nodes:

1. choose a size range (`lowBound`)
2. search the smallest subtree larger than `lowBound` in the tree
3. cut of that subtree, store it somewhere
4. start from step 2
5. schedule independend subtrees on different nodes
6. establish communication between nodes as far as necessary

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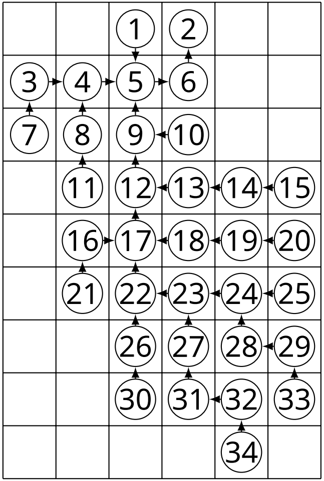
Tree Decomposition

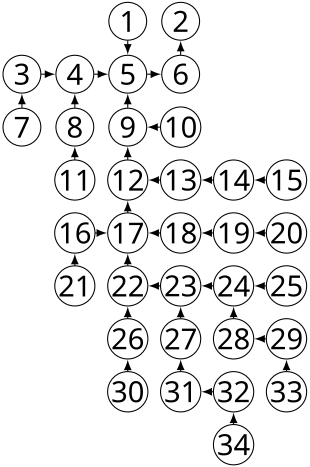
Goal



example: `lowBound`$=3$

Tree Decomposition

Goal



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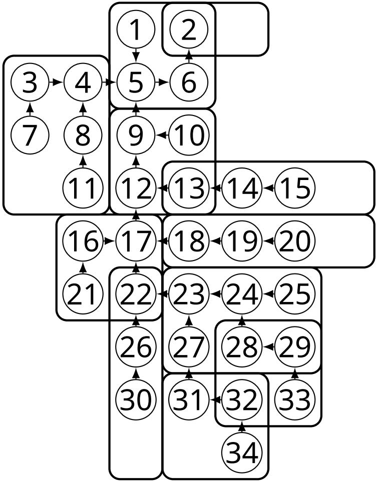
Tree Decomposition

Goal



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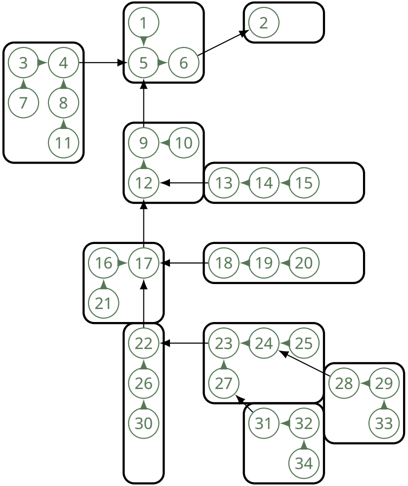
Tree Decomposition

Goal



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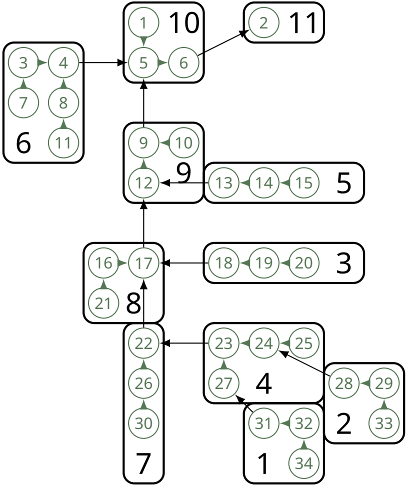
Tree Decomposition

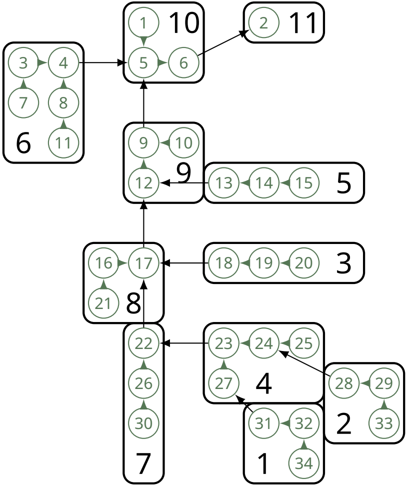
Goal



example: `lowBound`$= 3$

Tree Decomposition

Goal

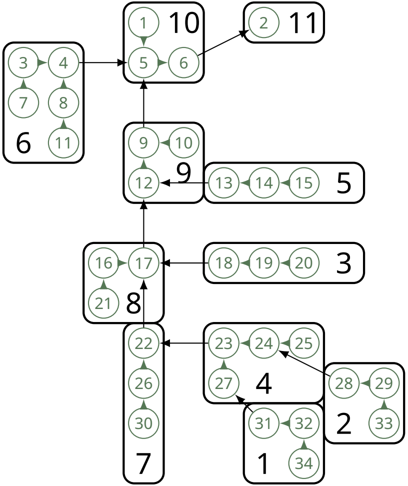


example: $\texttt{lowBound} = 3$

	timeslots						
process 1:	1	4	7	8	9	10	11
process 2:	2	5					
process 3:	3	6					

Tree Decomposition

Goal



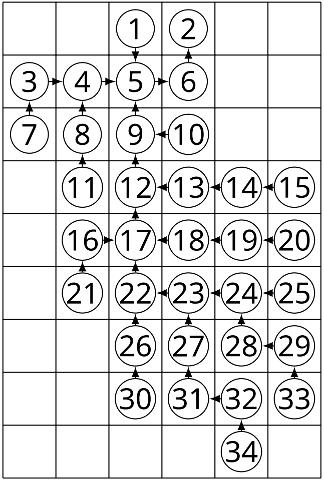
example: `lowBound`$= 3$

	timeslots						
process 1:	1	4	7	8	9	10	11
process 2:	2	5					
process 3:	3	6					

This is the shortest schedule, we can get with this subtrees: The *tree depth* is 7, we can not have a schedule shorter than the tree depth

Tree Decomposition

Tree data structure: basic info



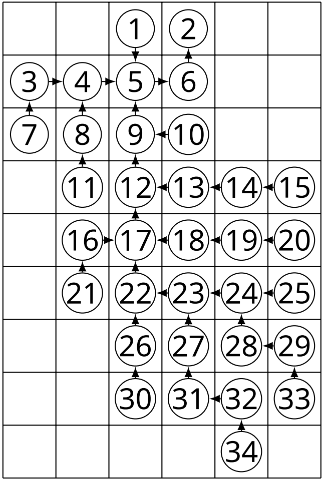
A classical tree data structure contains:

- `post`: a pointer to the parent tree node
- `Nprae`: the number of children
- `prae`: an array of pointers to the children

(note: each node is also a tree)

Tree Decomposition

Tree data structure: specific for tree decomposition



Specific data for the tree decomposition

- `siz`: the size of the tree
- `sizUp`: the size of the smallest subtree larger than `lowBound`
- ST: a pointer to metadata, if the tree node is the root node of a subtree

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Tree Decomposition

Tree data structure: derived type

```
type ptrTreeNode
  type(treeNode), pointer :: tN
end type ptrTreeNode
type treeNode
  type(ptrTreeNode)                          :: post
  integer(i4)                                :: Nprae
  type(ptrTreeNode),dimension(:),allocatable :: prae
  integer(i4)                                :: siz
  integer(i4)                                :: sizUp
  type(subtreeNode), pointer                 :: ST
end type treeNode
```

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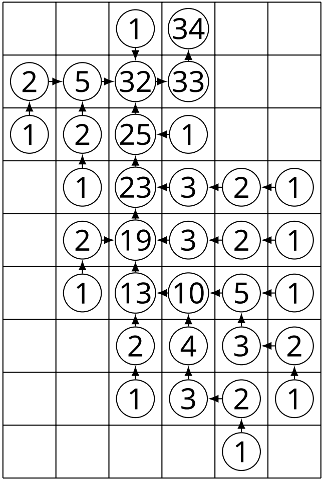
Tree Decomposition

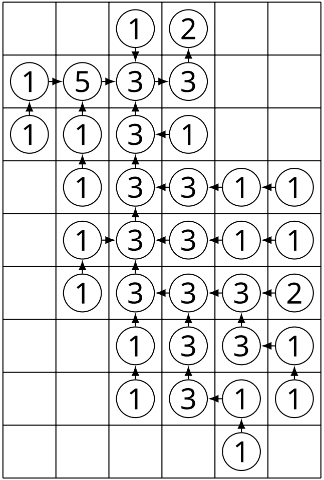
Tree data structure: Set siz for each node



- initialize `siz` with 1 for each tree node
- run though tree in routing order
- for each tree node:
  - for all its children add the value of `siz` of each child to own value of `siz`

Tree Decomposition

Tree data structure: Set sizUp for each node



- initialize `sizeUp` with 1 for each tree node
- run though tree in routing order
- for each tree node:
  - for all its children check, if `sizUp` has already been set for at least one child
  - if so, set `sizUp` of current tree node to the smallest of that values of its children
  - if not, check if `siz`≥`sizUp`
  - if so, set `sizUp`=`siz`
  - else `sizUp`=1 (this has to be set, so the subroutine can update the tree after a subtree gets cut of)

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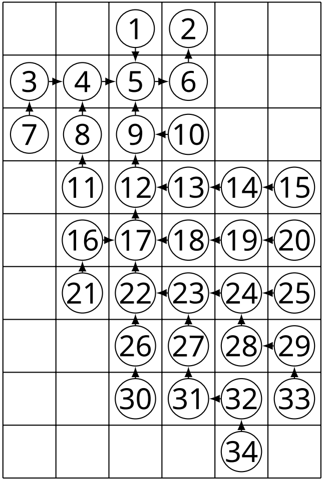
Tree Decomposition

Cut Of A Subtree

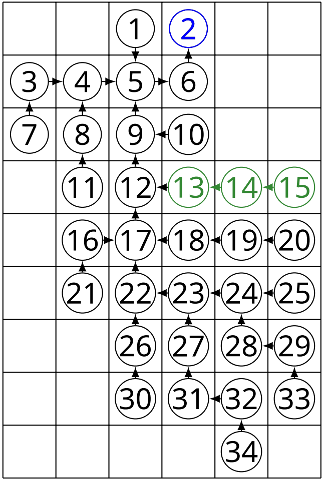


cut of a subtree in sublinear time (depth of tree)
Main idea, follow the branch with the smallest subtree (find_`branch`)
[Thomas H. Cormen, 2009, Harder, 2018]
start as root:

1. if one of the children of the current tree node has `sizUp`> 1
2. then switch to the child with the smallest value for `sizUp`, go to step 1
3. else cut of subtree at that node

Tree Decomposition

Cut Of A Subtree

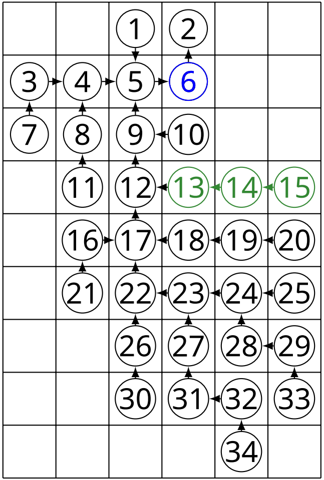


find_branch example:

1. tree node 2 has one child 6
2. 6 has sizUp$>$ 1 therefore we move to 6

Tree Decomposition

Cut Of A Subtree

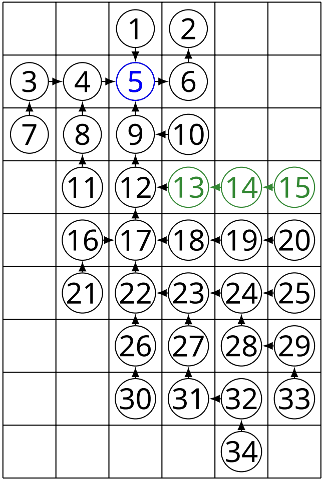


find_branch example:

1. tree node 6 has one child 5
2. 5 has sizUp$>$ 1 therefore we move to 5

Tree Decomposition

Cut Of A Subtree

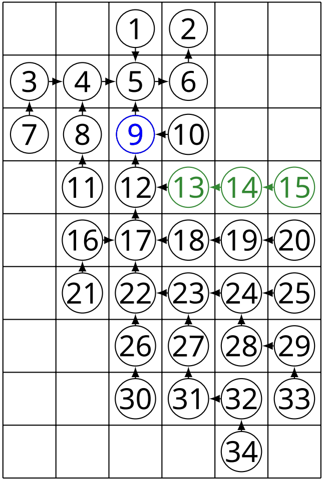


find_branch example:

1. tree node 5 has three childen 1, 4, 9
2. 1 has sizUp=1 (means not set), 4 has sizUp= 5, 9 has sizUp= 3, therefore we move to 9

Tree Decomposition

Cut Of A Subtree

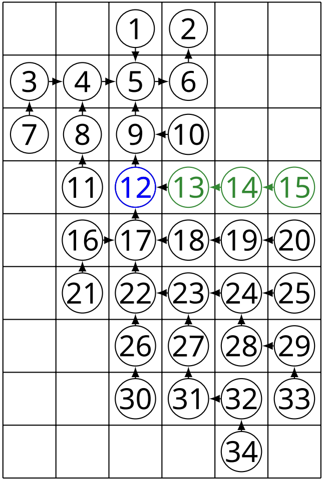


find_branch example:

1. tree node 9 has two childen 10, 12
2. 10 has sizUp=1 (means not set), 12 has sizUp= 3, therefore we move to 12

Tree Decomposition

Cut Of A Subtree

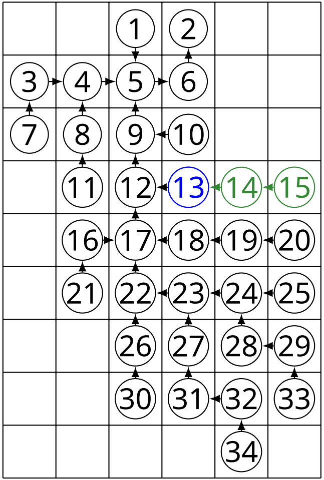


find_branch example:

1. tree node 12 has two childen 13, 17
2. 13 and 17 both have sizUp= 3, therefore we move to 13

Tree Decomposition

Cut Of A Subtree

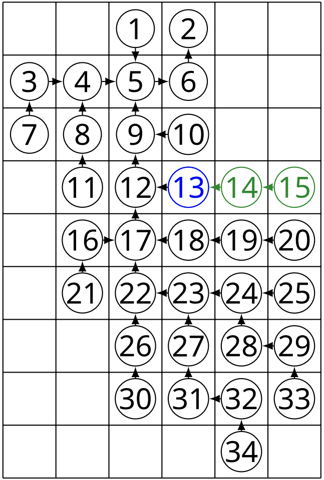


find_branch example:

1. tree node 13 has one child 14
2. sizUp values of all children of 13 are unset, therefore we cut of 13

Tree Decomposition

Cut Of A Subtree

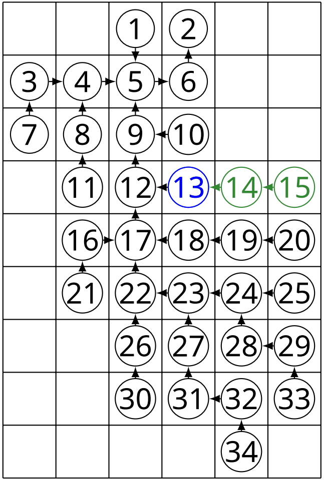


cut of a subtree

- return a pointer to the subtree root (13)
- `update_sizes`
- `initiate_subtreetreenode`
- in the parent node:
  - switch the cut of child with the last child in the `prae` array
  - reduce `Nprae` by one
- `update_tree`

Tree Decomposition

Cut Of A Subtree

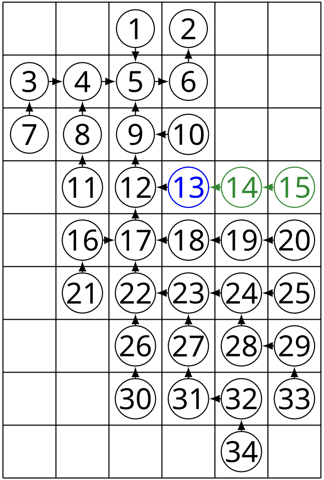


`update_sizes`:
for the cut of subtree (13) with size `redSize` (3)

1. if not root, move to parent
2. reduce `siz` of curret node by `redSiz`, go to 1

Tree Decomposition

Cut Of A Subtree

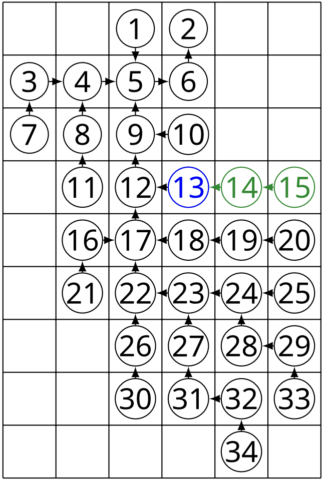


`initiate_subtreetreenode:`

- associate and allocate pointer ST of tree node
- set size of subtreetreenode to current size of tree node (after updating the tree structure, it is the correct size of the subtree)
- initialize other meta data with 0 and nullpointers

Tree Decomposition

Cut Of A Subtree

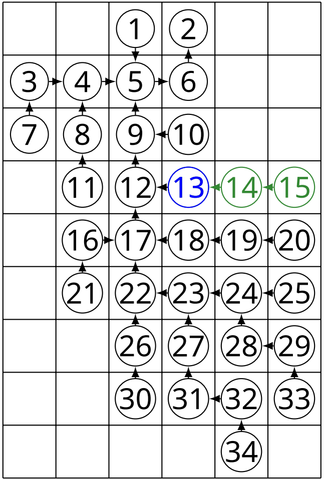


`update_tree`:
start at cut of tree node

1. update `sizUp` (as it was done before)
2. if not root, go to parent, go to step 1

Tree Decomposition

Cut Of A Subtree

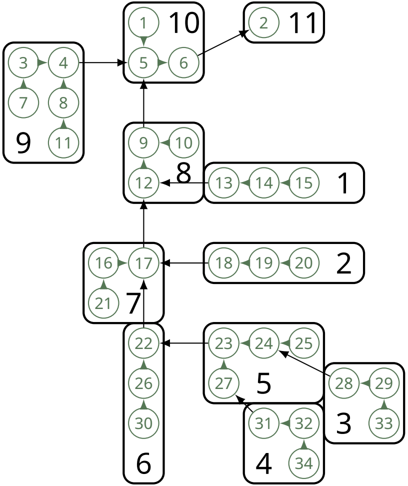


Special cases in the processess of cutting of subtrees:
root has no parent to be updated, so it has to be handled separately

- if one of its children has `sizeUp`>1, follow branch as usual
- else, cut of root

Tree Decomposition

Tree Decomposition

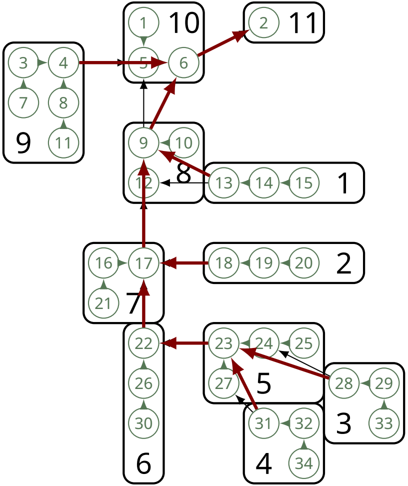


`decompose`

- as long as the last cut of subtree is not root
  - find and cut of subtree, return pointer to subtree
  - write pointer into an array

Tree Decomposition

Tree Decomposition

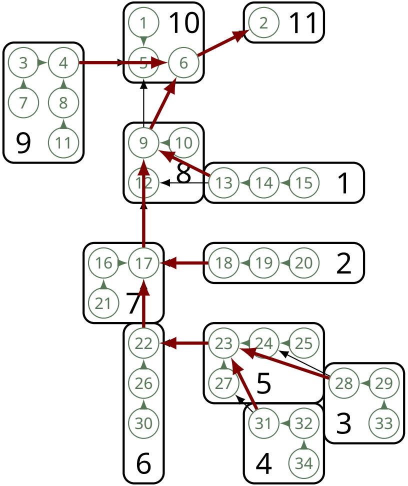


decompose

- as long as the last cut of subtree is not root
  - find and cut of subtree, return pointer to subtree
  - write pointer into an array
- set metadata of subtree nodes appropriately (set pointer to parents and children)

Tree Decomposition

The Subtree Data Structure



each subtreetree node has an associated pointer to derived type `subtreeNode` with classical tree data structure...

- `postST`: a pointer to the parent tree node (points to tree node)
- `NpraeST`: number of (subtreetree) children
- `praeST`: array of pointers to the (subtreetree) children

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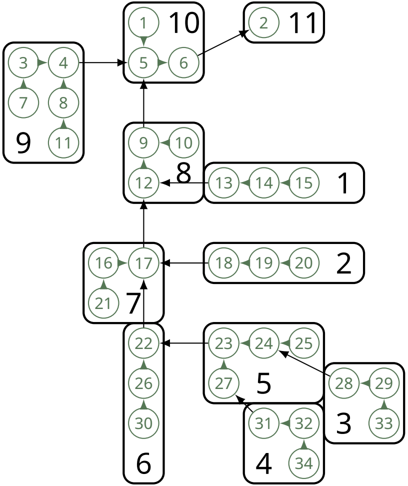
Tree Decomposition

The Subtree Data Structure



...and specific information for scheduling

- `levelST`: an array saving the distance to the root node and the distance to the farthest leaf in the subtreetree structure for scheduling purposes

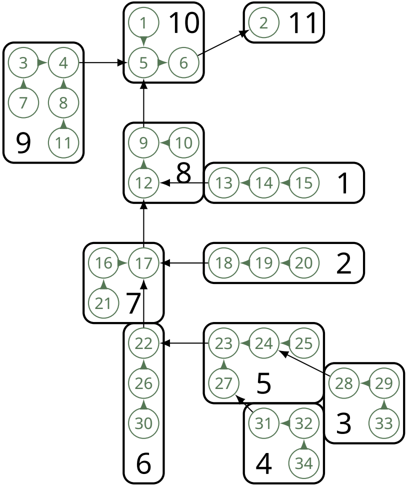
Tree Decomposition

Tree data structure: derived type

```
type treeNode
  ... (s.o.)
  type(subtreeNode), pointer                 :: ST
end type treeNode
type subtreeNode
  type(ptrTreeNode)                          :: postST
  integer(i4)                                :: NpraeST
  type(ptrTreeNode),dimension(:),allocatable :: praeST
  integer(i4)                                :: sizST
  integer(i4), dimension(2)                  :: levelST
end type subtreeNode
```

Tree Decomposition

Tree decomposition: scheduling



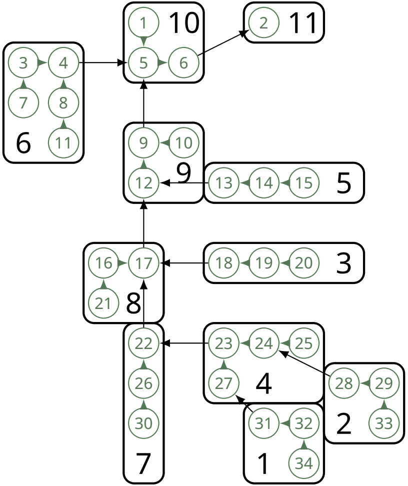
difference between two scheduling methods:

round robin

	timeslots							
process 1:	1	4			7		10	
process 2:	2		5			8		11
process 3:	3			6	9			

Tree Decomposition

Tree decomposition: scheduling



difference between two scheduling methods:

Hu's algorithm[Hu, 1961, Cheng and Sin, 1990]

	timeslots						
process 1:	1	4	7	8	9	10	11
process 2:	2	5					
process 3:	3	6					

MPI

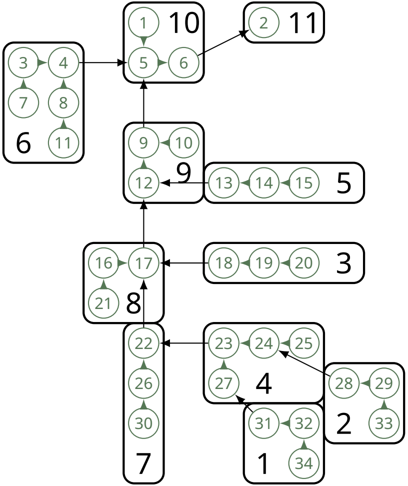
MPI

MPI

- n processes run the program
- each process knows its own rank and the number of processes
- in our case, process with rank 0 is the master process and cooridnates
- each process has its own memory, data has to be exchanged via *message sending interface* (MPI)

Tree Decomposition

Data Exchange Between Computing Nodes: Another Data Structure



tree data structures mainly constist of pointers that refer to physical memory adresses. Sending this data across nodes does not help. Solution:

- save the inices of the grid cells into an array in routing order, where the subtrees lay together
- save a `toNode` array for the links/edges

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MPI

basic structure

if process has rank 0:

- decompose tree
- prepare array in routing order and toNode array
- send subarrays and corresponding toNodes to the other processes
- send indices of leaves which are connected to subtree roots to processes
  - collect data from root nodes of the subtrees and send it to corresponding leaf nodes of adjacent subtree
- in the end: recollect subarrays

else:

- receive subarray, toNodes, and node indices of corresponding subtrees
  - collect input data for some leaves
  - do routing
  - send root output data to master
- in the end: send subarray back

MPI

Does It Work?

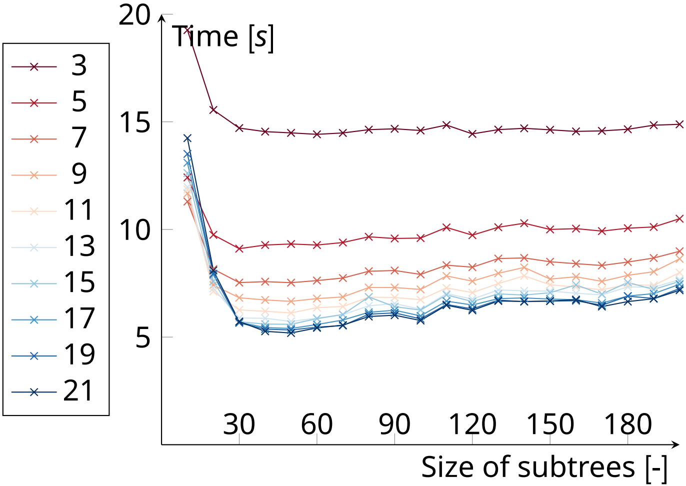
No, not that easily.
Communication needs more time than routing through arrays of size 10000.
A representative basin, Donau, has 26507 cells.
Ideas:

- do routing several times in a subtree, send array of data instead of one value
- or parallelize only independent trees with MPI

We are lucky: the first idea works

MPI

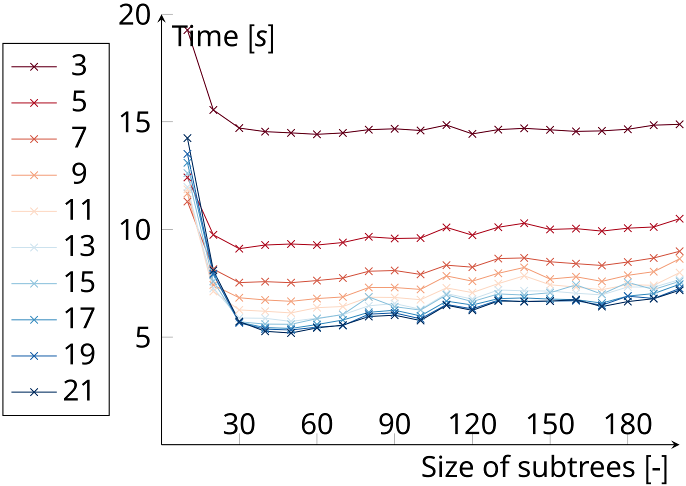
Times



- the basin tested, is the donau with 26507 cells
- collect arrays of 1000 data points before sending
- time is measured every 1000 steps (so 1000×1000 routing per time measurement)

MPI

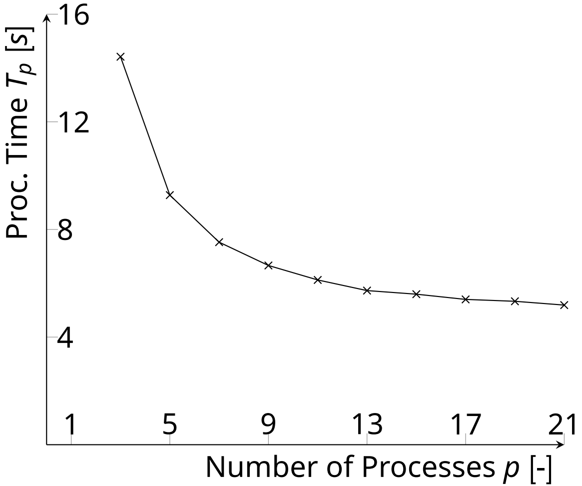
Times



- cutting into subtrees with sizes smaller than 50 results in communication overhead, means communication between the nodes takes more time than routing 1000 times through a subtree

MPI

Times



- for each number of processes the *y*-value is the minimum of the curves above
- more processors still make it faster but it does not tend to zero

OpenMP

OpenMP

differences to MPI

- arrays are not send over the network, but exchanged via shared memory
  - we have to handle data race problems
- sorting of subtrees to threads is not done by us but dynamically:
  - the routing through a subtree is assigned to a task
  - a waiting CPU gets a task/subtree when available

OpenMP

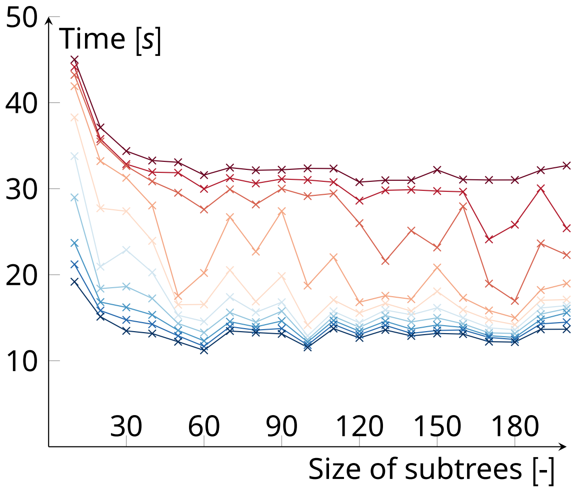
some code

```
!$OMP parallel private(rank) shared(testarray)
!$OMP single
call routing(root,testarray)
!$OMP end single
!$OMP barrier
!$OMP end parallel
```

```
recursive subroutine routing(root,array)
   ...
   do jj=1,root%tN%Nprae
      !$OMP task shared(root,array)
       call routing(root%tN%ST%prae(jj),array)
      !$OMP end task
   end do
   !$OMP taskwait
   if (associated(root%tN%post%tN)) then
        tNode=root%tN%post%tN%ind
        !$OMP critical
        array(tNode)=array(tNode)+array(root%tN%ind)
        !$OMP end critical
     end if
end subroutine routing
```

OpenMP

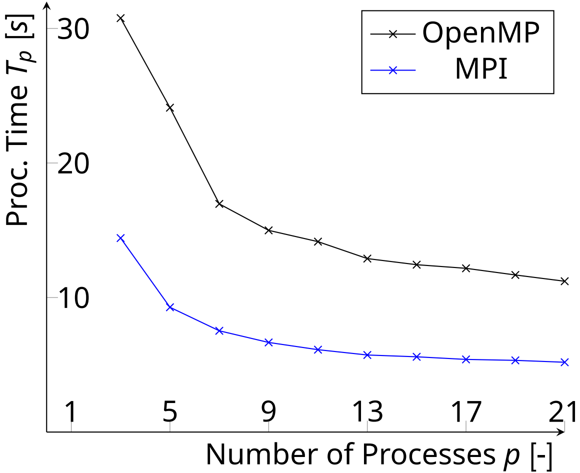
Times



- the basin tested, is again the donau with > 22000 cells
- collect arrays of 1000 data points before writing into shared array
- time is measured every 1000 steps (so 1000×1000 routing per time measurement)

OpenMP

Times



- for each number of processes the y-value is the minimum over `lowBound`

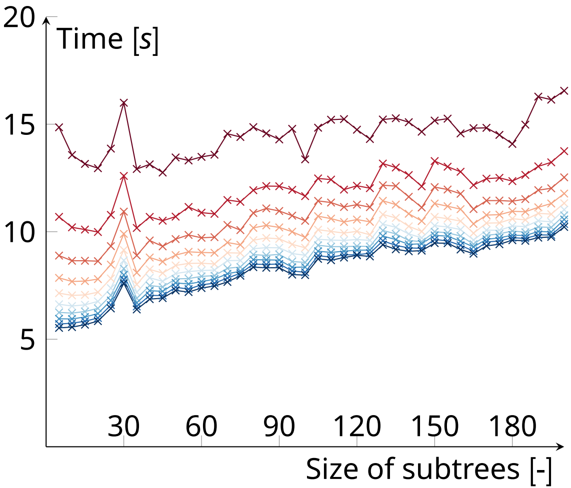
OpenMP

Times, reasons MPI is faster

- code with OpenMP tasks compiles poorly with gnu, intel is better
- Tasks are not sorted by priority. The tree is not a binary tree and it is unbalanced. (Approach: Use newer version of OpenMP, currently reading more literature)

OpenMP

Times, compiled with Intel



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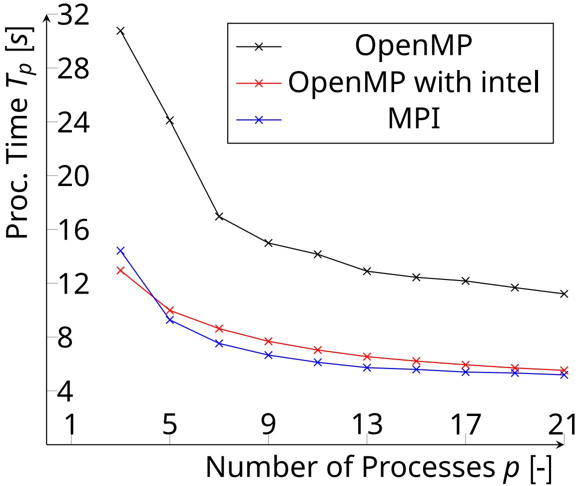
OpenMP

Times



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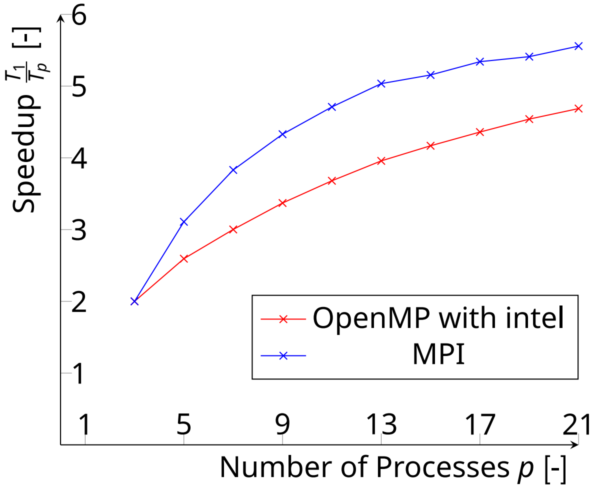
OpenMP

Times



- Speedup is the sequential time devided by the processing time with p processors $\frac{T_1}{T_p}$.
- Best case szenario is $\frac{T_1}{T_p} = p$
- We will never reach this because of the lower bound given by the tree depth

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